Ultrafast, intense laser pulse diagnostics (Lecture 1 of 2)

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Main research topics

**Laser-wakefield acceleration**
- Self-injection mechanisms
- X-ray and γ-ray generation
- Radiobiology with laser-driven electrons

**Ultraintense laser-solid interactions**
- Protons/light ion acceleration
- X-ray diagnostic development
- Laser-plasma interaction characterization

**Laser development**

For further infos: http://ilil.ino.it

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Summary of ILIL laser performances (upgrade Q3/2013)

Laser in figures
- energy: up to 450mJ on target
- pulse duration < 40fs
- ASE contrast > 10^9
- final beam diameter ~ 43mm
- \( M^2 < 1.5 \)
- rep rate: 10Hz

Focusing optics
Long focal length OAP
- \( f/#: 11.5 \)
- \( W: 16.6 \, \mu m \)
- \( I_{\text{max}}: 2 \times 10^{18} \, W/cm^2 \)
- \( Z_R: 620 \, \mu m \)
("LWFA" experiments)

Short focal length OAP
- \( f/#: 3.5 \)
- \( W: 5 \, \mu m \)
- \( I_{\text{max}}: 2 \times 10^{19} \, W/cm^2 \)
- \( Z_R: 56 \, \mu m \)
("TNSA" experiments)

2\textsuperscript{nd} order autocorrelation
ILILsubPW upgrade 2016-2017

Laser in figures
- energy: up to 3J on target
- pulse duration 30fs
- 100TW power
- ASE contrast > $10^9$
- rep rate: 1Hz

Final amplifier room

New Target Area
(before radioprotection bunker construction)
Outline

Lecture 1 of 2

A (not so short) introduction to the mathematical description of the temporal behaviour of ultrashort laser pulses (terminology, basic facts, ...)

- Spectral amplitude and phase
- Dispersion, dispersion compensation

Lecture 2 of 2

Experimental techniques for the temporal characterization of ultrashort laser pulses

- Photodiodes, streak camera
- 1st and 2nd order autocorrelators

- Advanced techniques for the pulse length and spectral phase measurements: FROG, SPIDER
- Contrast measurement techniques (in brief)

- Transverse functions characterization and wavefront correction
  - Wavefront characterization techniques
  - Wavefront correction and beam focusing
Mathematical description of the temporal behaviour of ultrashort pulses: basic facts

At a fixed point in space, for a linearly polarized pulse, the electric field can be simply written as

$$E(t) = A(t) \cos(\Phi_0 + \omega_0 t)$$

The field envelope and the pulse intensity are related by the expression ($A$ in V/m, $I$ in W/cm²)

$$A(t) = \sqrt{\frac{2}{\varepsilon_0 c}} \sqrt{I(t)} = 27.4 \sqrt{I(t)}$$

**ϕ₀**  Absolute phase, or **carrier envelope phase** (CEP)

Adding a time dependent phase function results in a so-called **chirped pulse**

$$\Phi(t) = \Phi_0 + \omega_0 t + \Phi_a(t)$$

An **instantaneous frequency** can be defined as

$$\omega(t) = \frac{d\Phi(t)}{dt} = \omega_0 + \frac{d\Phi_a(t)}{dt}$$

As we will see, there is no way to directly characterize the pulse duration (and phase) in the time domain (that is, to “directly” measure, for instance, $E(t)$), so that we need to think also to the frequency domain for a complete description.
Mathematical description of the temporal behaviour of ultrashort pulses: Introducing the spectral amplitude and phase

Using Fourier analysis, the field and its Fourier transform can be written as

\[ E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{i\omega t} \, d\omega \quad \text{and} \quad \tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t) e^{-i\omega t} \, dt \]

Of course, the knowledge of one of the two description is enough to completely characterize the pulse.

Most often, the so called “analytic signal” is used.

Being \( E(t) \) real, its Fourier transform is a Hermitian function: \( \tilde{E}(\omega) = \tilde{E}^*(-\omega) \)

This means that the knowledge of the Fourier transform for positive frequencies is enough to fully retrieve the signal.

We can thus define, for convenience, a new function in the frequency domain, retaining only the positive part of the FT:

\[ \tilde{E}^+(\omega) = \tilde{E}(\omega) \quad \text{for} \ \omega \geq 0 \]

\[ 0 \quad \text{for} \ \omega < 0 \]

and the corresponding FT\(^{-1}\)

\[ E^+(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}^+(\omega) e^{i\omega t} \, d\omega \]

Mathematical description of the temporal behaviour of ultrashort pulses: Introducing the spectral amplitude and phase

According to the above observation, $E^+(t)$, which is called analytic signal*, is enough to retrieve the “real” field $E(t)$. Indeed, it can be easily demonstrated that $E(t) = 2 \Re \{E^+(t)\}$ (Hint: define a similar function $E(\omega)$ in the frequency domain, only retaining the negative spectral part, identify the FT of $E(t)$ as the sum of $E^+$ and $E^-$, transform back to the time domain, ...)

$E^+(t)$ is a complex function, so that it can be written as

$$E^+(t) = |E^+(t)|e^{i\Phi(t)} = |E^+(t)|e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)} \propto \sqrt{I(t)}e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)} \propto A(t)e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)}$$

where the meaning of the different parameters is the same as above.

We will most often use the analytic signal in the following (sometimes, the + sign will be omitted for convenience).

*see L. Mandel, E. Wolf, *Optical Coherence and Quantum Optics* for an explanation
Mathematical description of the temporal behaviour of ultrashort pulses: Introducing the spectral amplitude and phase

Similarly, in the spectral domain we can introduce a *spectral amplitude* and a *spectral phase* as

$$\tilde{E}^+(\omega) = |\tilde{E}^+(\omega)| e^{-i\phi(\omega)} \propto \sqrt{I(\omega)} e^{-i\phi(\omega)}$$

What do the spectral amplitude and phase mean?

Spectral amplitude: proportional to the square root of $I(\omega)$, the usual “spectrum” as measured by a spectrometer

The spectral phase is basically the phase of each frequency in the waveform

What is the effect in the time domain? Two examples

$$\phi(\omega_i) = 0$$

$$\phi(\omega_i) = (i - 1)\frac{\pi}{5}$$
Mathematical description of the temporal behaviour of ultrashort pulses: Time vs frequency domain descriptions

**Time domain**

\[ E(t) = A(t)e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)} \]

**Frequency domain**

\[ \tilde{E}^+(\omega) = |\tilde{E}^+(\omega)|e^{-i\phi(\omega)} \propto \sqrt{I(\omega)e^{-i\phi(\omega)}} \]
Mathematical description of the temporal behaviour of ultrashort pulses: What does the spectral phase mean?

Actually, in the two examples two slides above we considered a linear dependence upon $\omega$.

In general, the spectral phase can be expanded into a Taylor series:

$$\phi(\omega) = \sum_{j=0}^{\infty} \frac{\phi^{(j)}(\omega_0)}{j!} \cdot (\omega - \omega_0)^j$$

where, of course, $\phi^{(j)}(\omega_0) = \frac{\partial^j \phi(\omega)}{\partial \omega^j} \bigg|_{\omega_0}$

This holds for a well-defined pulse. Basically, it means that each term in the expansion produces a pulse broadening or distortion that is significantly smaller than that of the previous term (see * for a deeper discussion on the optical meaning).

$$\frac{\partial \phi}{\partial \omega} \bigg|_{\omega_0} (\omega - \omega_0) \gg \frac{1}{2!} \frac{\partial^2 \phi}{\partial \omega^2} \bigg|_{\omega_0} (\omega - \omega_0)^2 \gg \frac{1}{3!} \frac{\partial^3 \phi}{\partial \omega^3} \bigg|_{\omega_0} (\omega - \omega_0)^3 \gg \ldots$$

Terminology: 2nd order term $\rightarrow$ Group Velocity Dispersion (GVD), 3rd order term $\rightarrow$ Third Order Dispersion (TOD)

Why introducing the spectral amplitude and phase?

A linear optical system acts on an input field by a multiplication by a (complex) transfer function in the frequency domain:

$$\tilde{E}_{out}(\omega) = \tilde{M}(\omega) \tilde{E}_{in}(\omega) = \tilde{R}(\omega) e^{-i\phi_d} \tilde{E}_{in}(\omega)$$

The spectral phase of the output pulse is thus modified according to

$$\phi_{in}(\omega) \mapsto \phi_{in}(\omega) + \phi_d(\omega)$$

An initially unchirped pulse ($\phi_{in}''(\omega_0) = 0$) can acquire a chirp if $\phi_d''(\omega_0) \neq 0$ (more details in a moment).

*see D.N. Fittinghoff et al., IEEE J. Sel. Top. Quant. Electr. 4, 430 (1998)
Spectral phase: the meaning of the first orders

Spectral phase expansion
\[ \phi(\omega) = \sum_{j=0}^{\infty} \frac{1}{j!} \phi^{(j)}(\omega_0)(\omega - \omega_0)^j \]

Reference pulse with \( \phi(\omega) = 0 \), so that \( \tilde{E}_{\text{ref}}(\omega) = |\tilde{E}_{\text{ref}}(\omega)| \) and \( E_{\text{ref}}^+(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\tilde{E}_{\text{ref}}(\omega)| e^{i\omega t} \, d\omega \)

Pulse with a constant (zero order) term
\[ \phi(\omega) = \phi(\omega_0) \]
On calculating the IFT, one gets
\[ E^+(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\tilde{E}^+(\omega)| e^{-i\phi(\omega_0)} e^{i\omega t} \, d\omega = e^{-i\phi(\omega_0)} E_{\text{ref}}^+(t) \]
This corresponds to acquiring an absolute phase \( \phi(\omega_0) \)

Pulse with a 1st order term
\[ \phi(\omega) = \phi'(\omega_0)(\omega - \omega_0) \]
On calculating the IFT, one gets
\[ E^+(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\tilde{E}_{\text{ref}}^+(\omega)| e^{-i\phi'(\omega_0)(\omega - \omega_0)} e^{i\omega t} \, d\omega = e^{i\phi'(\omega_0)\omega_0} \tilde{E}^+(t - \phi'(\omega_0)) \]
This corresponds to a time shift of the pulse, with
\[ T_g = \phi'(\omega_0) \]

To summarize: constant and linear terms in the spectral phase have no effects on the pulse duration.
Higher order terms and pulse duration

For a pulse with a given bandwidth (and spectrum), the shortest duration is reached when no chirp occurs; in the frequency domain, this translates into the spectral phase exhibiting a constant or linear dependence upon $\omega$.

We start calculating the pulse duration for a general pulse as

$$\Delta t^2 = \int_{-\infty}^{+\infty} (t - \langle t \rangle)^2 I(t) \, dt = \int_{-\infty}^{+\infty} \langle (t - \langle t \rangle) E(t) \rangle^2 \, dt$$

Using the Plancherel’s identity and the equation aside

$$\mathcal{F}[(t - \langle t \rangle) E(t)] = ie^{-i\omega(t)} \frac{\partial}{\partial \omega} \left(e^{i\omega(t)} \tilde{E}(\omega)\right)$$

(easy to demonstrate)

and finally, on introducing the spectral amplitude and phase and calculating the derivative

$$\Delta t^2 = \int_{-\infty}^{+\infty} \left| \frac{\partial}{\partial \omega} |\tilde{E}(\omega)|^2 \right|^2 \, d\omega + \int_{-\infty}^{+\infty} \left| \tilde{E}(\omega) \right|^2 \left| \frac{\partial}{\partial \omega} (\omega \langle t \rangle - \phi(\omega)) \right|^2 \, d\omega$$

The first integral is ever positive and depends upon the spectral amplitude (or, the spectrum). As for the second one:

$$\frac{\partial}{\partial \omega} (\omega \langle t \rangle - \phi(\omega)) = \langle t \rangle - \phi'(\omega_0) - \frac{\partial}{\partial \omega} \text{(spectral phase terms } O((\omega - \omega_0)^2))$$

We saw above that the second term accounts, in the time domain, for a pulse delay, $T_g = \phi'(\omega_0)$ so that the first two terms cancels out.

Thus, a further (positive) contribution to the time duration exists if the spectral phase exhibits higher order terms (GVD, TOD, ...).

Ideally, in order to keep a pulse as short as possible, one thus look for optical elements which do not transfer quadratic phase to the pulse or (most of times!) for devices for adjusting/compensating for the accumulated spectral phase.

See also L. Walmsley et al., Rev. Sci. Instr. 72, 1 (2001)
Spectral phase modifications and time duration: a few examples

Unchirped (bandwidth limited) pulse, constant spectral phase

Unchirped pulse (bandwidth limited), shifted in time due to a linear (negative) spectral phase

Figure credits: Springer Handbook of Laser Optics, (editor F. Trager), 2007
Spectral phase modifications and time duration: a few examples

Symmetrically broadened pulse, due to a 2nd order term in the spectral phase

3rd order spectral phase term, leading to quadratic group delay. In the time domain, oscillations appear before or after the main pulse, depending on the sign of the 3rd order
Spectral phase modifications and time duration: a few examples

\[ \pi \text{ step in the spectral phase at } \omega = 0 \]

Sine modulation to the spectral phase

Therefore, acting on the spectral phase the time profile of the pulse can be shaped ("pulse shaping")
Summary of the effects of spectral phase modifications to the time behaviour.

- **Initial pulse**
- **Case of propagation in vacuum**
  - Effect of first and second order dispersion
  - Effect of first and third order dispersion

**Group delay** gives the retardation of the pulse in time: $GD$
Recall that
\[ \frac{d\phi_m}{d\omega} = \frac{L}{c} \left( n + \omega \frac{dn}{d\omega} \right) = \frac{L}{c} \left( n - \omega \frac{dn}{d\omega} \right) \]
\[ \phi''_m = \frac{d^2\phi_m}{d\omega^2} = \frac{L}{c} \left( 2 \frac{dn}{d\omega} + \omega \frac{d^2n}{d\omega^2} \right) \]

If \( \frac{dn}{d\omega} \) is not equal to zero (dispersion), each frequency will move with a different velocity and the pulse gets broadened (spectral phase wise, this results in a 2nd order not null).

For ordinary transparent media in the visible region, normal dispersion is encountered (\( \frac{dn}{d\omega} > 0 \)), which results in positive chirp (lower wavelengths arrive before higher ones).
How to modify the spectral phase on purpose: gratings stretcher/compressor and Liquid Crystal Spatial Light Modulators

As you know, the usage of grating elements allow 2nd order (and higher!) dispersion to be introduced, with both down (usually in a compressor) and up-chirping (usually in a stretcher)

How to modify the spectral phase on purpose: Acousto-Optic Programmable Dispersive Filter

Geometry of an acousto-optic programmable dispersive filter

Pulse shaping is achieved via interaction of co-propagating acoustic and optical waves in a birefringent photoelastic medium

Acoustic wave produced by a (programmable) RF generator

Two optical modes can be coupled efficiently by acousto-optic interaction only in the case of phase matching

At each point, each acoustic frequency couples the two modes for only a given (optical) frequency

If the phase velocity is different for the two modes, arbitrary delays can be imposed
How to modify the spectral phase on purpose: Acousto-Optic Programmable Dispersive Filter
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Experimental techniques for the temporal characterization of ultrashort laser pulses

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Advanced techniques for the pulse length and spectral phase measurements: FROG, SPIDER

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Transverse functions characterization and wavefront correction

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Measurement of the pulse profile using “fast” detectors: photodiodes

Direct photoelectric effect, with photon energy larger than the bandgap of the semiconductor

Typical electrical signal given by a “fast” photodiode in response to a femtosecond pulse

![Graph showing the typical electrical signal given by a photodiode.](image)

The finite time required to completely move all the free carriers induced by the laser field prevents the measurement of the pulse envelope

Using ultrasmall active areas (~\(\mu\text{m}^2\)) and high reverse voltages, response times of the order of ~10ps can be reached.
Measurement of the pulse profile using “fast” detectors: streak camera

Current streak-camera response in the visible range is of the order of a few 100fs

Advanced techniques, based on (auto)correlation methods, are needed in order to measure the pulse duration of ~10fs pulses
Basic ingredients common to ALL the pulse measurement methods

1. Time-space transformation. A given delay is obtained by letting the pulse to be delayed travel longer paths; fs delays require (variable) micron-scale optical path lengths, which can be safely produced and measured using current technology translation stages and optical encoders.

2. Use of the (auto)correlation functions to retrieve the pulse behaviour.

Given two fields $E_{\text{ref}}(t)$ and $E(t)$, the measurement of their 1st order correlation function

$$G_1(\tau) = \int_{-\infty}^{+\infty} E_{\text{ref}}^*(t) E(t - \tau) \, dt$$

allows $E(t)$ to be recovered provided that $E_{\text{ref}}(t)$ (reference pulse) is fully known.

If a reference pulse is not available, more advanced methods must be employed.

In what follows, detectors with response times much longer than the pulse duration are considered, so that basically they measure the pulse energy:

Read signal $\propto \int_{-\infty}^{+\infty} |E(t)|^2 \, dt \propto \int_{-\infty}^{+\infty} I(t) \, dt \propto$ pulse energy
Characterization of the temporal behaviour of a laser pulse using a reference pulse

**Time-domain interferometry**

A scan of a sufficiently large delay (>pulse duration) is carried out, and the signal corresponding to each delay is recorded.

\[
S(\tau) \propto \int_{-\infty}^{+\infty} |E_{\text{ref}}(t - \tau) + E(t)|^2 \, dt \\
= \int_{-\infty}^{+\infty} |E_{\text{ref}}(t - \tau)|^2 \, dt + \int_{-\infty}^{+\infty} |E(t)|^2 \, dt + \left(\int_{-\infty}^{+\infty} E(t) E_{\text{ref}}^*(t - \tau) \, dt + \text{c.c.}\right)
\]

Notice that the last two terms correspond to the 1st order correlation function. Taking the Fourier Transform, one gets

\[
\mathcal{F}(S)(\omega) = A\delta(\omega) + \hat{E}(\omega) \hat{E}_{\text{ref}}^*(\omega) + \hat{E}(-\omega) \hat{E}_{\text{ref}}(-\omega)
\]

from which the spectral phase of the pulse can be retrieved provided that the reference pulse is completely characterized.

Recall that the spectral amplitude is simply related to the spectrum

\[
\hat{E}^+(\omega) = |\hat{E}^+(\omega)| e^{-i\phi(\omega)}
\]

Characterization of the temporal behaviour of a laser pulse using a reference pulse

Frequency-domain interferometry

\[
S(\omega) \propto \left| \mathcal{F}(E_{ref}(t-\tau) + E(t)) \right|^2 = \left| \tilde{E}_{ref}(\omega)e^{i\omega\tau} + \tilde{E}(\omega) \right|^2 \\
= \left| \tilde{E}_{ref}(\omega) \right|^2 + \left| \tilde{E}(\omega) \right|^2 + \left( \left| \tilde{E}_{ref}(\omega) \tilde{E}^*(\omega) \right| e^{-i(\phi_{ref}(\omega) - \phi(\omega))} e^{i\omega\tau} + c.c. \right) \\
= \left| \tilde{E}_{ref}(\omega) \right|^2 + \left| \tilde{E}(\omega) \right|^2 + 2 \left| \tilde{E}_{ref}(\omega) \right| \left| \tilde{E}^*(\omega) \right| \cos[\omega\tau - (\phi)_{ref}(\omega) - \phi(\omega)]
\]

Interference fringes appear in the power spectrum with an average fringe spacing inversely proportional to the time delay

The delay is kept at a fixed value, and the spectrum of the overlapped pulses is measured

The phase of the fringe pattern yields the spectral phase difference between the reference and the unknown pulse

Main issue with these correlation techniques: a **completely** characterized reference pulse (with a spectrum larger than the one of the pulse to be measured) is usually not available!